

Thouless, Haldane, Kosterlitz



**Nobel 2016**

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**Sherbrooke, 7 novembre 2016**



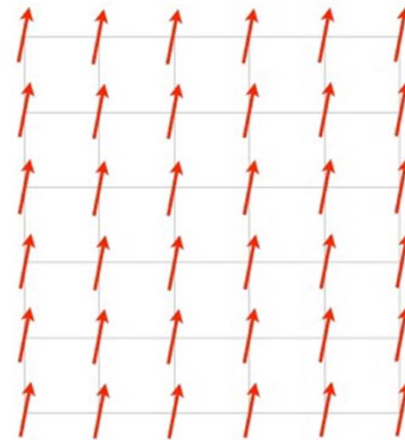
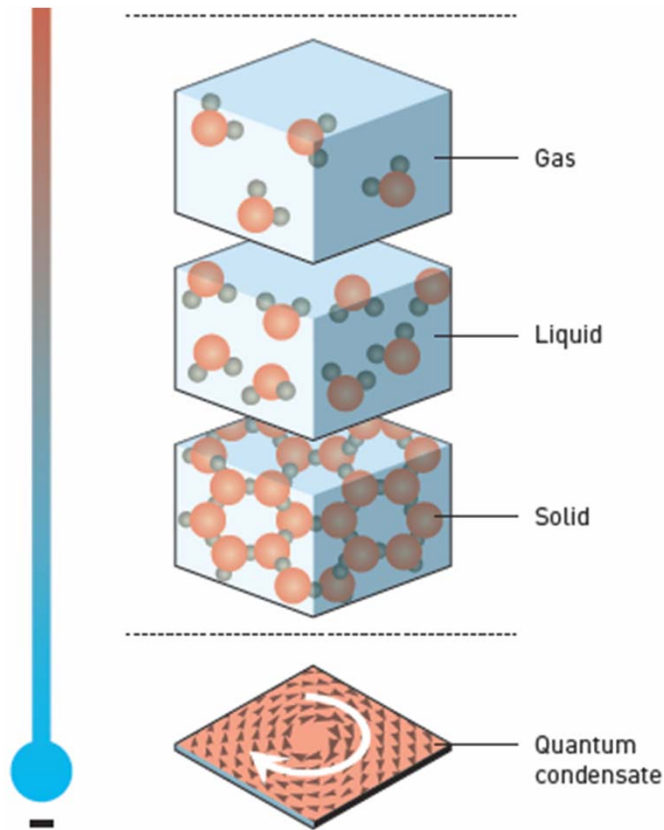
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# Annonce du comité Nobel

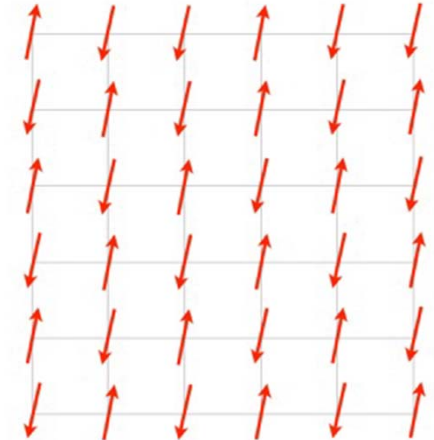
- *Pour les découvertes théoriques de transitions de phase topologiques et de phases topologiques de la matière.*
- 1972 transition Kosterlitz-Thouless
  - Certains types d'aimants et les films supraconducteurs
- 1982 Thouless *et al.* effet Hall quantique (Ion)
- 1983 Haldane: Effets quantiques topologiques sur les chaînes de spin (David S.)
- Informatique quantique (David P.)



# États, symétrie brisée, émergence, rigidité



Ferromagnet

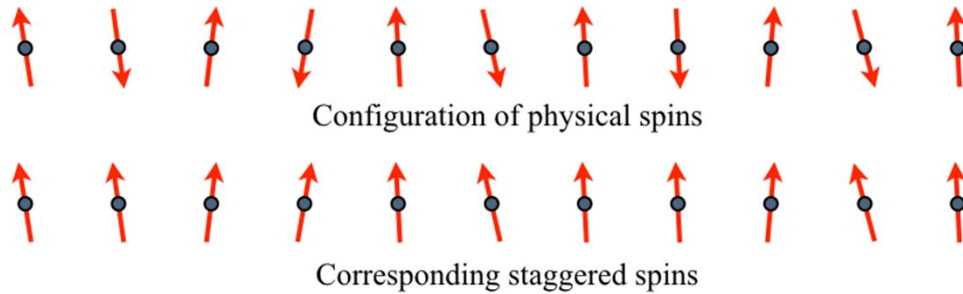


Antiferromagnet

Illustration: ©Johan Jarnestad/The Royal Swedish Academy of Sciences



# Paramètre d'ordre



$$\vec{m}_i = \mu(-1)^i \vec{S}_i$$

Supraconducteur

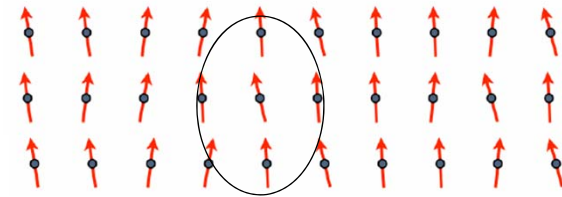
$$\psi = \sqrt{\rho_s} e^{i\theta}$$



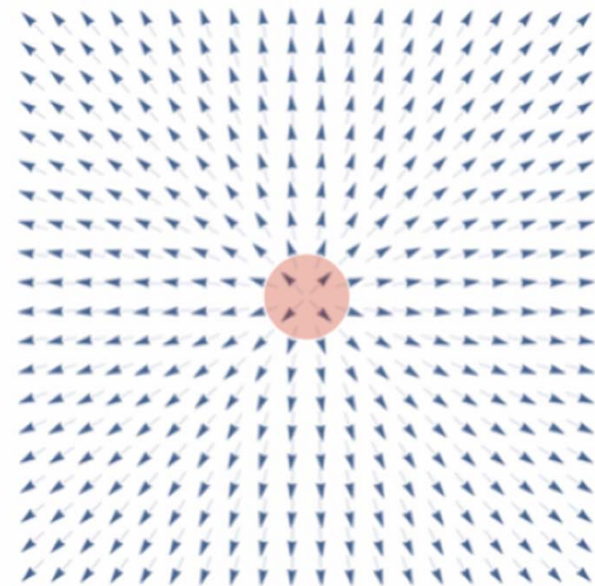
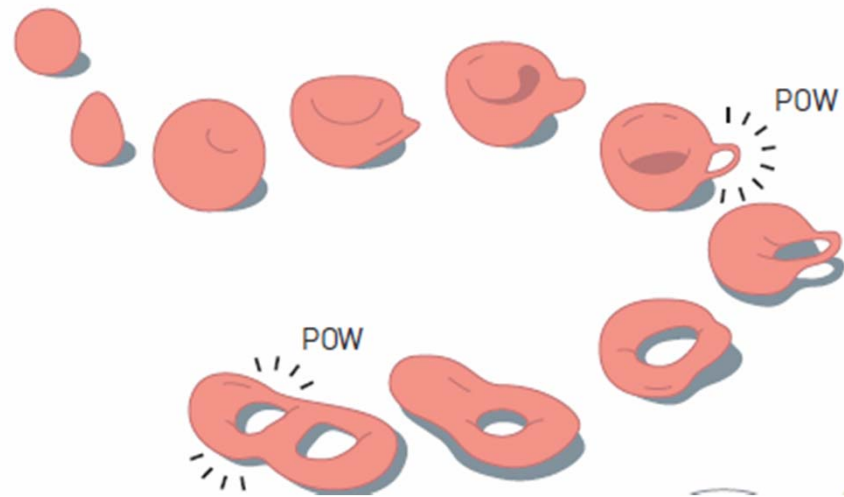
# Aimant plan facile ou supraconducteur $d=2$

## Topologie 1

$$\frac{1}{2\pi} \oint_C d\vec{r} \cdot \vec{\nabla} \theta(\vec{r})$$



$$(\theta_1 - \theta_2) + (\theta_2 - \theta_3) + (\theta_3 - \theta_1) = 0$$



$$\frac{1}{2\pi} \oint_C d\vec{r} \cdot \vec{\nabla} \theta(\vec{r}) = \pm 1$$

# Transition de phase

$$T < T_c$$

$$\lim_{\mathbf{r} \rightarrow \infty} \langle m(\mathbf{r})m(\mathbf{0}) \rangle = m_0^2$$

$$T > T_c$$

$$\lim_{\mathbf{r} \text{ grand}} \langle m(\mathbf{r})m(\mathbf{0}) \rangle = C e^{-r/\xi} \underset{=_{\mathbf{r} \rightarrow \infty}}{0}$$

$$T = T_c$$

$$\lim_{\mathbf{r} \text{ grand}} \langle m(\mathbf{r})m(\mathbf{0}) \rangle = C' \frac{1}{r^{1+\eta}} \underset{=_{\mathbf{r} \rightarrow \infty}}{0}$$



# La transition Kosterlitz-Thouless: topologie 2

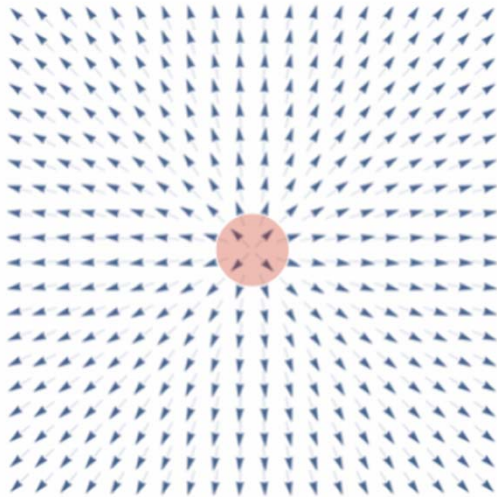
$$H_{XY} = \frac{J}{2} \int d^2r (\vec{\nabla}\theta(\vec{r}))^2$$

$$\langle e^{i(\theta(\vec{r})-\theta(\vec{0}))} \rangle \sim \left(\frac{a}{r}\right)^{\frac{k_B T}{2\pi J}}$$

$$\lim_{r \text{ grand}} \langle m(\mathbf{r})m(\mathbf{0}) \rangle = C' \frac{1}{r^{1+\eta}} \underset{r \rightarrow \infty}{=} 0$$



# Et les tourbillons?



$$H_{XY} = \frac{J}{2} \int d^2r (\vec{\nabla} \theta(\vec{r}))^2$$

$$E_v = \frac{J}{2} \int d^2r \left( \frac{1}{r} \right)^2 = J\pi \ln \frac{L}{a}$$





# La transition Kosterlitz-Thouless: topologie 2

$$E_v = \frac{J}{2} \int d^2r \left( \frac{1}{r} \right)^2 = J\pi \ln \frac{L}{a}$$

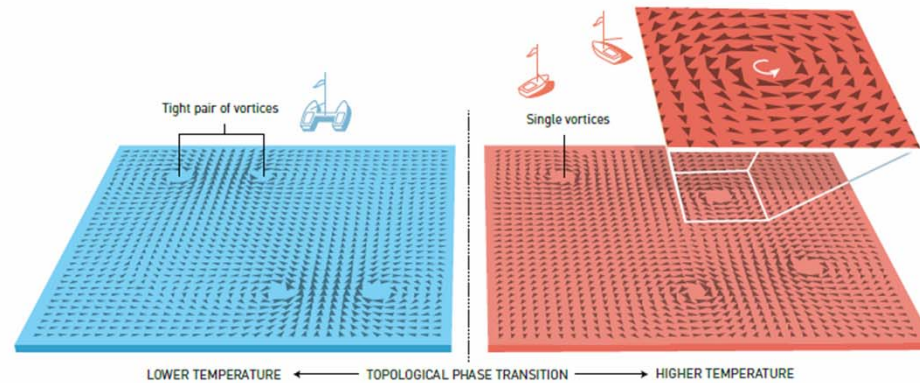


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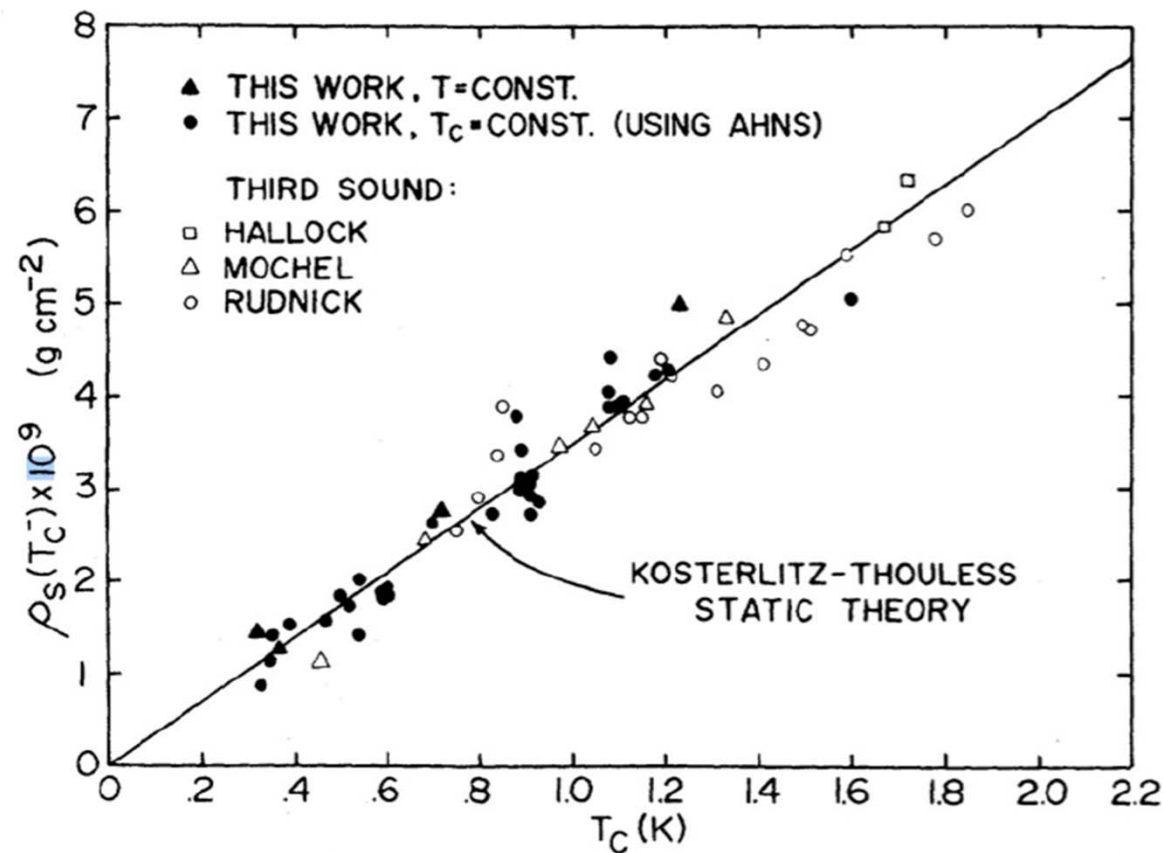
$$F = E - TS = J\pi \ln \left( \frac{L}{a} \right) - T k_B \ln \left( \frac{L^2}{a^2} \right)$$

$$\mathbf{min} (F) = \mathbf{min} (E - T S)$$



# Vérification expérimentale d'une prédiction

$$\rho(T_c) = T_c \frac{2}{\pi} \frac{m^2 k_B}{\hbar^2}$$



# Un peu partout

- Films de supraconducteurs désordonnés
- Réseau de jonctions Josephson
- Supraconducteurs granulaires
- Monocouches solides

*Cas quantique*

*$T = 0$*

*$\min E(\text{Pression}, \text{Spin})$*

